Celera MOTION
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What Motor Torque Constant to Use for Drive Type – Theory and Application

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INTRODUCTION

Calculating motor torque from available drive current can be confusing due to many different drive types and the multiple ways current is specified ($I_{pk-sine}$, I_{DC}, I_{RMS}). This paper will provide the key formulas for torque constant and motor current, from fundamental principles of three-phase motor theory. It will also walk through the many ways torque can be calculated, using a per-phase torque, phase-to-phase torque constant, and sinusoidal torque constant, while applying the appropriate currents based on drive type (trapezoidal or sine).

THREE PHASE MOTOR THEORY

The motor theory described in this paper is based on a brushless motor (BLAC or permanent magnet synchronous machines) configuration that has a sinusoidal back EMF waveform for each phase-to-neutral winding. Assumptions are as follows: torque angle curves and current waveforms are assumed to be pure sinusoids of the same amplitude. For equations using the per-phase torque constant, this assumes that input current is supplied to the neutral (center tap) connection.

When applying current to a singular motor phase of a BLAC (phase-to-neutral), these types of motors generate sinusoidal torque waveforms versus electrical angle (as the shaft rotates mechanically). The electrical angle is N/2 multiplied by (*) mechanical angle, where N is the number of poles of the motor.

Figure 1 below shows a graphical drawing for the phasor representation of torque angle curve, with a three-phase brushless motor. The three vectors created by $K_{t_{\emptyset_x}} * i_x$ are shown as dotted lines, meant to represent that these vectors scale up and down in magnitude, as they rotate around the phase diagram, always staying 120 degrees apart. The magnitude of each vector varies based on current i_x (constant for trapezoidal commutation, and sinusoidal for sine commutation) and the sinusoidal variation of the per-phase torque constant ($K_{t_{\emptyset_x}}$) with rotor position.

In this derivation, we are making the assumption that the $K_{t_\emptyset}$ values are equal in magnitude (actual variations are typically small, but for the purposes of these derivations, we are stating there are no variations), and separated by 120 electrical degrees.

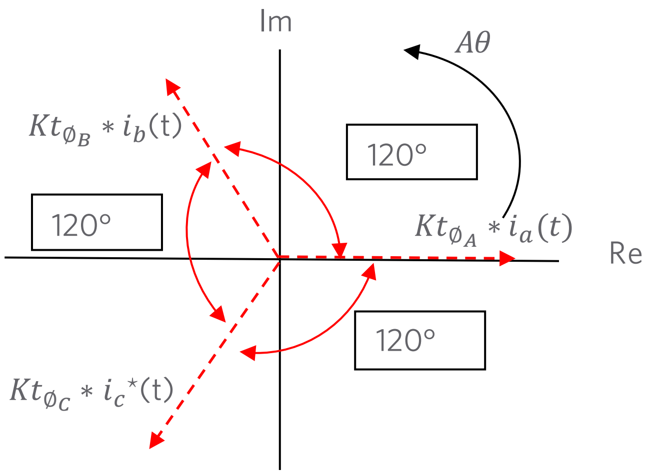

Figure 1 – Spatial Variation of Torque in the Re/Im plane

TRAPEZOIDAL DRIVE

Trapezoidal commutation allows current to only flow through two of the three motor phases at any given time, and the current flowing is considered to be DC (see Figure 2). This results in $i_a(t) = -i_c(t)$ & $i_a(t) = I_{DC}$. For the purposes of the derivation, we are ignoring the ramp up and ramp down of the current based on the time constant (RL) of the motor phases. With this configuration, i_x values within the phasor diagram of Figure 1 are constant in time, and have no variation (unlike sinusoidal drive, which will be discussed below).

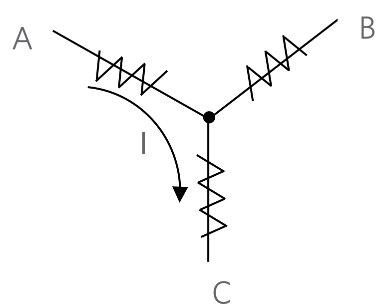

Figure 2 – Trapezoidal Current in Three-Phase Motor

The general equation for motor torque is a function of the single-phase motor torque and the current flowing through each of the three phases, represented as follows:

Equation (1)

$$T_m = K_{t_{\emptyset}} * [i_a(t) * \cos(A\theta) + (i_b(t) * \cos(A\theta - 120) + (i_c(t) * \cos(A\theta - 240)]$$

based on stated assumption of $Kt_{\emptyset} = Kt_{\emptyset_A} = Kt_{\emptyset_B} = Kt_{\emptyset_C}$

As stated above for trapezoidal drives, current is only flowing in two of three phases at any one time, so the math reduces significantly, and the motor torque ($T_{m_{trap}}$) can be solved algebraically or by using vector math as shown in Figure 3.

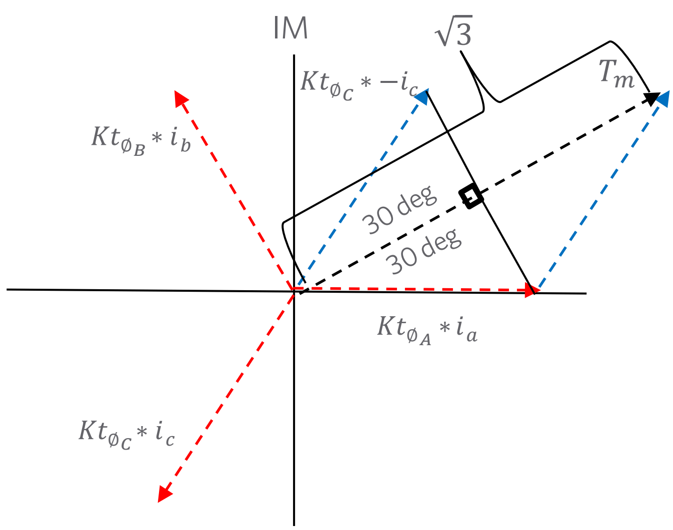

Figure 3 – Vector Math for Trapezoidal Commutation

Solving Equation (1) algebraically requires angle $A\theta$ to be known. From the vector diagram of Figure 3, we can see that when current is only flowing in phase A and out phase C, the value is 30 degrees, and we arrive at Equation (2) below.

Equation (2)

$$T_{m_{trap}} = K_{t_{\emptyset}} * (I_{DC} * \cos(30^o) + I_{DC} * \cos(30^o))$$

Solving for torque:

Equation (3)

$$T_{m_{trap}} = K_{t_\emptyset} * I_{DC} * \left(2 * \frac{\sqrt{3}}{2}\right) = \sqrt{3} * K_{t_\emptyset} * I_{DC}$$

Since current I_{DC} is only ever flowing in two phases at a time, an equivalent two-phase torque constant is then derived:

$$T_{m_{trap}} = K_{t_{\emptyset\emptyset_{trap}}} * I_{DC}$$

Where:

Equation (4)

$$K_{t_{\emptyset\emptyset_{trap}}} = \sqrt{3} * K_{t_\emptyset}$$

Equation (4) yields the result of the two-phase motor torque constant, as a function of the single-phase.

SINUSOIDAL DRIVE

Sine drives use a commutation scheme that flows current in all three phases at all times, with the sum always equaling zero ($I_A + I_B + I_C = 0$). Different than a trapezoidal drive, the currents i_x are now modulating sinusoidally as follows:

Equation (5)

$$i_a(t) = I * \cos(\omega t)$$

Equation (6)

$$i_b(t) = I * \cos(\omega t - 120^o)$$

Equation (7)

$$i_c(t) = I * \cos(\omega t - 240^o)$$

$$where\ \omega t\ rotates\ with\ A\theta\ or\ A\dot{\theta}t, and\ \omega = A\dot{\theta}$$

Substituting Equations (5), (6), and (7) into Equation (1) yields:

Equation (8)

$$T_{m_{sine}} = K_{t_\emptyset} * I[\cos(\omega t)\cos(A\theta) + \cos(\omega t - 120)\cos(A\theta - 120) + \cos(\omega t - 240)\cos(A\theta - 240)]$$

Solving Equation (8) for where $where\ \omega t = A\theta$ and $A\theta = 0°$ yields:

Equation (9)

$$T_{m_{sine}} = K_{t_{\emptyset}} * I_{pk-sine} * \frac{3}{2}$$

Like Equation (3) above, which shows torque developed as a function of the single-phase torque constant and the current flowing in the motor, Equation (9) can be re-written to represent a sinusoidal torque constant. Equation (10) yields the equivalent torque constant for current flowing when using a sine drive.

Equation (10)

$$K_{t_{sine}} = \frac{3}{2} * K_{t_{\emptyset}}$$

BACK EMF CONSTANT AND TORQUE CONSTANT

In Wye connected motors, the center tap (neutral) is rarely available, making phase-to-neutral measurements impossible. The best thing to do is work within the constraints and make phase-to-phase measurements, then work backwards through the equations to arrive at per-phase, phase-to-phase, and sine torque constants.

When working in SI units, the per-phase $K_{e_{\emptyset}}$ constant is equal to the single-phase torque constant $K_{t_{\emptyset}}$. In a three-phase system, if we could measure a single-phase voltage, we would find that the relationship between single-phase and phase-to-phase voltage is:

Equation (11)

$$V_{bemf_{\emptyset\emptyset}} = \sqrt{3} * V_{bemf_{\emptyset}}$$

This is because K_e is directly correlated to V_{bemf}

Equation (12)

$$K_e = \frac{V_{bemf}}{kRPM}$$

Given the relationship between K_e and voltage shown in Equation (12), the voltage output can be substituted for K_e in Equation (11), yieling Equation (13).

Equation (13)

$$K_{e_{\emptyset\emptyset}} = \sqrt{3} * K_{e_{\emptyset}}$$

In Equation (4) above, we show how $K_{t_{\emptyset\emptyset_{trap}}} = \sqrt{3} * K_{t_{\emptyset}}$ and when combined with $K_{e_{\emptyset}} = K_{t_{\emptyset}}$ we get:

Equation (14)

$$K_{e_{\emptyset\emptyset}} = K_{t_{\emptyset\emptyset}}$$

The two-phase representation $K_{t_{\emptyset\emptyset}}$ is then equivalent to $K_{t_{\emptyset\emptyset_{trap}}}$.
Therefore, Equation (13) is also equal to Equation (14).

COMPARING TRAPEZOIDAL WITH SINE COMMUTATION

Since measurment of $K_{e_{\emptyset\emptyset}}$ (and subsequent conversion to $K_{t_{\emptyset\emptyset_{trap}}}$) invloves two phases at a time, it is convenient to find a conversion from $K_{t_{\emptyset\emptyset_{trap}}}$ to $K_{t_{sine}}$.

We know that $T_{m_{sine}} = T_{m_{trap}}$, therefore using Equations (3) and (9), solving for I_{DC} we see that:

Equation (15)

$$I_{DC} = 0.866 * I_{pk-sine}$$

and

Equation (16)

$$K_{t_{\emptyset\emptyset_{trap}}} * I_{DC} = K_{t_{sine}} * I_{pk}$$

Substituting Equation (15) above into Equation (16), yields Equation (17).

Equation (17)

$$K_{t_{\emptyset\emptyset_{trap}}} * 0.866 * I_{pk-sine} = K_{t_{sine}} * I_{pk-sine}$$

Simplifying the equation then yields Equation (18).

Equation (18)

$$K_{t_{\emptyset\emptyset_{trap}}} * 0.866 = K_{t_{sine}}$$

It should be noted that referring to $K_{t_{sine}}$ as a line-to-line or phase-to-phase, torque constant, is generally not good pratice because it implies only two phases are working at once – which is only true for very brief moments when the phase-to-phase current of a single-phase pair passes through zero. It is useful, however, to scale between a two-phase ($K_{t_{\emptyset\emptyset_{trap}}}$) and a sinusoidal ($K_{t_{sine}}$) torque constant, as many drives and motor companies spec current (I_{DC} or $I_{pk-sine}$) and torque constant ($K_{t_{\emptyset\emptyset_{trap}}}$ or $K_{t_{sine}}$) differently.

EXAMPLES: LAB TEST DATA

Test Components:

- Agility™ Series slotless motor, zero cogging, specifications from datasheet:

English units	SI
$K_e = 4.1\, {}^{V}/_{kRPM}$	$K_e = 0.039\, {}^{V}/\frac{rad}{sec}$
$K_{t_{sine}} = 4.67\, {}^{oz-in}/_{A_{pk-sine}}$	$K_{t_{sine}} = 0.034\, {}^{Nm}/_{A_{pk-sine}}$
$K_{t_{\emptyset\emptyset_{trap}}} = 5.5\, {}^{oz-in}/_{A_{DC}}$	$K_{t_{\emptyset\emptyset_{trap}}} = .039\, {}^{Nm}/_{A_{DC}}$

- Everest XCR servo drive (sine commutation)
- Vibrac Torque Sensing Device
- Oscilloscope (output has 1:1 conversion between V and oz-in)
- PC for MotionLab 3

Static Kt Test Versus Ke to Kt Calculation

1. Back drive motor and calculate $K_{e_{\emptyset\emptyset}} = V_{pk-sine}/\frac{rad}{sec}$
2. Convert $K_{e_{\emptyset\emptyset}}$ to $K_{t_{sine}}$ and $K_{t_{\emptyset\emptyset_{trap}}}$
3. Apply various current outputs (0.1, 0.2, 0.2, and 0.3 $A_{pk-sine}$) from the drive, using the drive's software (MotionLab 3)
4. Hold motor shaft in place long enough to get full reading on oscilloscope that is reading torque
5. Record oscilloscope output in volts and convert to oz-in
6. Calculate 4 values of $K_{t_\emptyset}$ (in units of ${}^{Nm}/{}_{A_{pk-sine}}$) using Equation (10).

$$T_{m_{sine}} = K_{t_\emptyset} * I_{pk-sine} * \frac{3}{2}$$

7. Use Average to calculate $K_{t_{sine}}$
8. Calculate Percent Error
9. Solve for $K_{t_{\emptyset\emptyset_{trap}}}$
10. Calculate Percent Error

Calculations

- $K_{t_\emptyset}$ **Calculation**

Calculating $K_{t_\emptyset}$ for measurements 1 and 2:

1. Current = 0.2 $A_{pk-sine}$
   T_m = 0.94 oz-in = 0.00664 Nm
   $K_{t_\emptyset}$ = 0.0221 ${}^{Nm}/{}_{A_{pk-sine}}$

2. Current = 0.2 $A_{pk-sine}$
   T_m = 0.92 oz-in = 0.00650 Nm
   $K_{t_\emptyset}$ = 0.0217 ${}^{Nm}/{}_{A_{pk-sine}}$

Then calculate the average:

Average $K_{t_\emptyset}$ = 0.0219 ${}^{Nm}/{}_{A_{pk-sine}}$

- Use Average $K_{t_\emptyset}$ to calculate $K_{t_{sine}}$

  $K_{t_{sine}}$ = 1.5 * $K_{t_\emptyset}$ = 1.5 * 0.0219 $^{Nm}/_{A_{pk-sine}}$ = 0.0329 $^{Nm}/_{A_{pk-sine}}$

- Calculate Percent Error

  Expected $K_{t_{sine}}$ from datasheet = 0.034 $^{Nm}/_{A_{pk-sine}}$

  Expected $K_{t_{sine}}$ from measured K_e = 0.033 $^{Nm}/_{A_{pk-sine}}$

  $\boldsymbol{Error\ (using\ measured\ K_e)} = \left(\frac{expected - measured}{expected}\right) * \mathbf{100} = \left(\frac{\mathbf{0.033 - 0.0329}}{\mathbf{0.033}}\right) * 100 =$ 0.3%

- Solve for $K_{t_{\emptyset\emptyset_{trap}}}$

  $Average\ K_{t_\emptyset}$ = 0.0219 $^{Nm}/_{A_{pk-sine}}$

  $K_{t_{\emptyset\emptyset_{trap}}}$ = $\sqrt{3}$ * 0.0219 = 0.038 $^{Nm}/_{A_{DC}}$

- Calculate Percent Error

  Expected $K_{t_{\emptyset\emptyset_{trap}}}$ from datasheet = 0.039 $^{Nm}/_{A_{DC}}$

  Expected $K_{t_{\emptyset\emptyset_{trap}}}$ from measured K_e = 0.038 $^{Nm}/_{A_{DC}}$

  $\boldsymbol{Error\ using\ measured\ K_e} = \left(\frac{expected - measured}{expected}\right) * \mathbf{100} = \left(\frac{\mathbf{0.038 - 0.039}}{\mathbf{0.038}}\right) * 100 = 2.6\%$

SUMMARY OF TEST DATA

For percent error calculations, the measured $K_{e_{\emptyset\emptyset}}$ was used as the 'expected' value, instead of the motor spec sheet value, due to variations of 10% allowed for magnet strength differences. The calculated

$K_{t_{sine}}$ with <1% error and the calculated $K_{t_{\emptyset\emptyset_{trap}}}$ of ~3% error are both within the acceptable range given the error of the measurements. The calulations above, conducted with a slotless motor, also eliminate any cogging torque from altering measured data. Cogging torque can have a huge impact on static torque testing and should always be considered. For this reason, back driving motors and using $K_{e_{\emptyset\emptyset}}$ to arrive at $K_{t_{\emptyset\emptyset_{trap}}}$ is the preferred approach. The static testing above is also a common method, but can quickly be made irrelevant when cogging is introduced.

CONCLUSION

The torque constant K_t is a critical motor parameter that determines the amount of output torque per input current. Because of the three-phase nature of brushless DC/AC/PMSM, and how they are driven, confusion quickly arises as to which K_t value to use when calculating output torque for a given input current. The two most common ways to state K_t are $K_{t_{\emptyset\emptyset_{trap}}}$ with units of Nm/A_{DC} and $K_{t_{sine}}$ with units of $Nm/A_{pk-sine}$. The derivation of both comes from per-phase motor theory and the conversion between them has been shown. Care must be taken to understand how the motor will be driven in terms of the drive type, and how the motor manufacturer is stating K_t. When testing to confirm stated K_t values, be cautious when cogging is present. For this reason, it is recommended to back drive motors to measure K_e and convert to $K_{t_{\emptyset\emptyset_{trap}}}$ and $K_{t_{sine}}$ as shown in this paper.

ACKNOWLEDGMENTS

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